A METHOD OF DETERMINING THE TEMPERATURE DEPENDENCE OF THE THERMAL CONDUCTIVITY OF DIELECTRICS AND SEMICONDUCTORS

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A method is described for determining the temperature behavior of the thermal conductivity of solid nonmetallic materials. Results of calculations based on this method are presented.

To determine the temperature behavior of the thermal conductivity of dielectrics and semiconductors in a single experiment, the following method may be used. Two identical specimens of the test material are located between a common heater and blocks with reference (known) heat capacities and are heated by parallel heat fluxes, each of which proceeds from the heater through the specimen to the reference (Fig. 1). We shall consider the heating rate to be constant. Part of the heat obtained by the reference blocks goes into heating the thermal insulation surrounding the references. By measuring the temperature difference between the references in a linear heating process, we can determine the temperature behavior of the thermal conductivity of the test material.

The references must have different heat capacities; for a large difference in heat capacity, it is convenient to use one solid and one hollow block of a light material, as in [1].

To derive the calculation relations we shall carry out an analysis of the temperature field in one of the specimens, to determine the temperature of contact with the solid reference block. We shall first examine the ideal case in which the thermal conductivity of the thermal insulation is negligibly small in comparison with that of the specimen. Then all the heat passing through the specimen goes into heating the reference. This situation also applies when there is an ideal jacket whose temperature is exactly equal to that of the reference.

The general heat-conduction equation and the boundary conditions have the form

$$\frac{\partial t_1(x,\tau)}{\partial\tau} = k_1\frac{\partial^2 t_1(x,\tau)}{\partial x^2}; \tag{1}$$

$$-\lambda_1 S_1\left[\frac{\partial t_1(x,\tau)}{\partial x}\right]_{x=l_1} = c_2 m_2\frac{dt_2}{d\tau},$$

$$t_1(x,0)=t_0,\quad t_2|_{\tau=0}=t_0,$$

$$t_1(0,\tau)=t_0+b\tau,\quad b=\mathrm{const}(\tau). \tag{2}$$

The solution of (1) by an operational method, after eliminating the exponential terms which decrease rapidly with time, gives the temperature field in the specimen the form

$$t_1(x,\tau)=t_0+b\left\{\tau-\left[\frac{c_2 m_2}{\lambda_1 S_1}+\frac{l_1}{k_1}\right]x-\frac{x^2}{2k_1}\right\}. \tag{3}$$

The temperature at the boundary of the specimen and the reference is

$$t_1(l_1,\tau)=t_0+b\left\{\tau-\frac{c_2 m_2 l_1}{\lambda_1 S_1}-\frac{c_1 m_1 l_1}{2\lambda_1 S_1}\right\}. \tag{4}$$

Then

$$\lambda_1=\frac{bl_1}{S_1\Delta t_1}\left(c_2 m_2+\frac{c_1 m_1}{2}\right). \tag{4a}$$

Writing (4) for the second specimen-reference pair, and taking the difference between the expressions, we find the temperature difference between the references, whence

$$\lambda_1=(c_2 m_2-\overset{*}{c_2}\overset{*}{m_2})bl_1/S_1\Delta t. \tag{5}$$

The expression obtained is similar to Yagfarov's formula [1] for the cylindrical case.

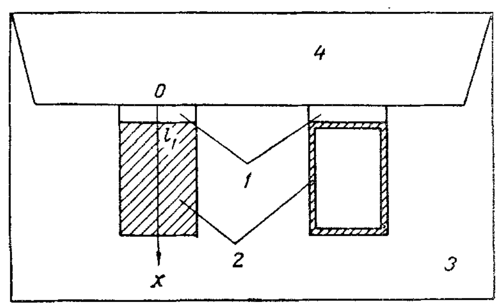

Fig. 1. Schematic of the method: 1) Specimen; 2) reference blocks; 3) thermal insulation; 4) heater.

Thus, the thermal conductivity of the test material in the absence of heat losses may be determined either from the temperature difference between the references, as in Yagfarov's cylindrical method, or from the temperature drop in the specimen. In the latter case one specimen and one reference are sufficient, but the specimen heat capacity must be known, albeit approximately.

We shall further examine a real case of thermal losses from the reference to the insulation, which occurs unavoidably in the plane model examined. With one specimen-reference pair these losses may be sharply reduced by use of a jacket surrounding the reference and in contact with the heater. However, the jacket does not fully eliminate heat exchange with the reference, and it is therefore necessary to introduce correction terms into the calculation formula.

We shall now explain the role of heat losses to the insulation in the presence of two specimens and two references.

We shall compute the heat loss from the reference to the insulation. We shall assume for simplicity that this occurs only through the end of the reference, and we shall take the area of the end to be the whole area of contact of the reference with the insulation.

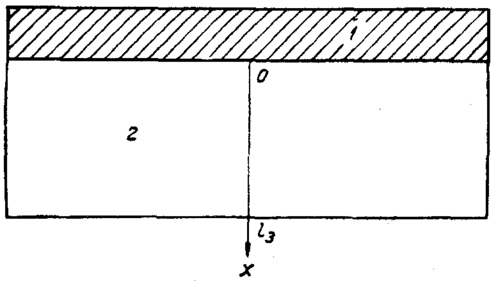

Fig. 2. Model when the temperature drop in the specimen is neglected: 1) Reference; 2) insulation.

We shall consider the heat flux to the insulating layer. For simplicity of calculation we shall assume that the temperature of the reference is equal to that of the heater, i.e., we shall neglect the temperature drop in the specimen (Fig. 2). This assumption is valid, since the thermal resistance of the specimen is much less than that of the insulating layer. For the insulating layer the heat conduction equation and the boundary conditions have the form

$$\frac{\partial t_3(x, \tau)}{\partial \tau} = k_3 \frac{\partial^2 t_3(x, \tau)}{\partial x^2},$$

$$t_3(0, \tau) = t_0 + b\tau, \quad t_3(l_3, \tau) = t_0, \quad t_3(x, 0) = t_0.$$

Solving this equation, we find the temperature field in the insulating layer to be

$$t_3(x, \tau) - t_0 = b\tau \frac{l_3 - x}{l_3} + b\left[\frac{(l_3 - x)^3}{6k_3 l_3} - \frac{l_3(l_3 - x)}{6k_3}\right].$$

Thus, the temperature in the insulator is distributed according to a cubic law.

The heat flux at the boundary of the reference and the insulator is

$$-\lambda_3 S_3 \frac{\partial t_3(x, \tau)}{\partial x}\bigg|_{x=0} = -\lambda_3 S_3 b\left[\frac{l_3}{6k_3} - \frac{\tau}{l_3} - \frac{l_3}{2k_3}\right].$$

We shall calculate the temperature field in the specimen, allowing for heat losses to the insulator. We return to Fig. 1 and Eq. (1). We replace boundary conditions (2) by the following:

$$-\lambda_1 S_1\left[\frac{\partial t_1(x, \tau)}{\partial x}\right]_{x=l_1} = c_2 m_2 \frac{dt_2}{d\tau} - \lambda_3 S_3 b\left[\frac{l_3}{6k_3} - \frac{\tau}{l_3} - \frac{l_3}{2k_3}\right].$$

The solution has the form

$$t_1(x, \tau) = t_0 + b\tau - b\left[\left(\frac{c_2 m_2}{\lambda_1 S_1} + \frac{l_1}{k_1}\right)x - \frac{x^2}{2k_1}\right] + b\frac{\lambda_3 S_3}{\lambda_1 S_1}\left\{\left[\frac{2k_1 l_1 c_2 m_2 + \lambda_1 S_1 l_1^2}{2k_1 l_3 \lambda_1 S_1} - \frac{l_3}{3k_3} - \frac{\tau}{l_3}\right]x - \frac{x^3}{6k_1 l_3}\right\}, \tag{6}$$

i.e., when there are heat losses to the insulator, the temperature in the specimen is not distributed according to a parabolic law (3), but according to a cubic law.

It may be seen from (6) that when there are heat losses, we can no longer determine λ_1 from the temperature drop in the specimen. If, however, we use a second specimen and a second reference (with another value of heat capacity), then the thermal conductivity of the specimen is quite simply expressed in terms of the temperature difference between the references. In fact, using (6), we find

$$\lambda_1 = (c_2 m_2 - c_2^* m_2^*)\frac{bl_1}{S_1 \Delta t}\left(1 - \frac{\lambda_3 S_3 l_1}{\lambda_1 S_1 l_3}\right). \tag{7}$$

The factor $(1 - \lambda_3 S_3 l_1/\lambda_1 S_1 l_3)$ is close to unity, and therefore the previous formula (5) is valid, if $l_3 \gg l_1$, which usually may be realized.

In cases in which the above factor differs appreciably from unity, it is not difficult to calculate the influence of heat loss to the insulation, in the final expression for λ_1. Replacing λ_1 by its value from (5) in the given factor, i.e., using the method of successive approximations, we obtain

$$\lambda_1 = (c_2 m_2 - c_2^* m_2^*) bl_1/S_1 \Delta t - K,$$

where

$$K = \lambda_3 S_3 l_1/S_1 l_3.$$

The quantity K is determined from calibration tests, it being necessary to determine K as a function of temperature. In the final expression for λ_1 we should also calculate the thermal resistances at the contacts of the specimen with the heater and the reference. Then

$$\lambda_1^0 = l_1/(l_1/\lambda_1 - R_K),$$

where R_K is the total thermal resistance at the contacts, determined experimentally. For dielectric materials the quantity R_K may usually be neglected.

It should be stressed that the insulation surrounding the left and right references must be identical in properties and dimensions.

Thus, in the proposed method the need to use a jacket drops out. The method is convenient for materials amenable to mechanical processing or pressure forming, from which identical specimens may easily be prepared.

The temperature range suggested is −150° to +1000° C.

At high temperatures this method has an advantage over other methods, since it excludes errors associated with the growth of heat losses at increased temperatures.

The method described, like that of Yagfarov, is based on the use of two identical specimens and two different references. However, in distinction from the Yagfarov method, the second specimen-reference pair serves here not only to exclude from the calculation formula the self-heat capacity of the specimen (which is inappreciable in the given case), but above all to exclude the influence of heat losses from the reference to the insulator, which are absent in the cylindrical Yagfarov arrangement.

Notation

$t_1(x, \tau)$—temperature at point x of specimen at time τ; λ_1, k_1, c_1, m_1—respectively, thermal conductivity, thermal diffusivity, specific heat, and mass of specimen; l_1—specimen thickness; S_1—specimen cross section area; b—heating rate; Δt_1—temperature drop in specimen; Δt—temperature difference between references; c_2, c_2^*—specific heats of references; m_2, m_2^*—masses of references; t_2—temperature of reference; $t_3(x, \tau)$, λ_3, k_3—respectively, temperature, thermal conductivity, and thermal diffusivity of insulation; S_3—area of contact of reference with insulation; l_3—thickness of insulating layer.

REFERENCE

1. M. Sh. Yagfarov, DAN SSSR, 127, no. 3, 1959.

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